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Geometry teaching practices of rural educators in Boyacá Colombia, interpretive analysis from curricular references 1991-2017

Julio Cesar Àvila Morales
julio.avila@uptc.edu.co
ORCID: 0009-0000-9269-6487
Universidad Pedagógica y Tecnológica de Colombia

Hugo Hernando Mendoza Vargas
hugo.mendoza@uptc.edu.co
ORCID: 0000-0003-3664-4505
Universidad Pedagógica y Tecnológica de Colombia

Abraham Rico Segura
abraham.rico@uptc.edu.co
ORCID: 0009-0005-9659-4169
Universidad Pedagógica y Tecnológica de Colombia

Abstract

The central axis of this article is to present an interpretive analysis from the teaching practices carried out by 8 educators from rural unitary schools and multigrade classrooms from different regions of the department of Boyacá, with the aim of strengthening the geometric reasoning competence, particularly the transition. from the two-dimensional to the three-dimensional and vice versa, analysis of the implementation of quality benchmarks and curricular improvement proposed by the Ministry of National Education. In addition, it seeks to contribute to the teaching process from the geometric reasoning criterion proposed by Van Hiele, specifically in relation to the development of the first three levels proposed for primary basic education students.

Keywords: geometric reasoning; curricular references; unitary schools and multigrade classrooms; plane and spatial geometry; primary basic education.

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Introduction

This article presents to the academic community an interpretative analysis of the teaching practices of two- and three-dimensional geometry, presenting findings obtained by the author, the result of the accompaniment as a tutor teacher for more than 10 years, in the program "Todos a Aprender", in which he has worked with teachers from rural institutions with unitary headquarters and multigrade classrooms in the department of Boyacá. and as a result of the research in the Doctoral Program in Education Sciences of the Pedagogical and Technological University of Colombia. With the analysis of the data obtained, the processes and main elements of didactic transposition that these teachers carry out in their teaching exercise are revealed. It also includes a synthesis of the public policies that have strengthened the curriculum, through the training of educators in Colombia, specifically in this type of classroom.

Through the multi-method or mixed methodology used in the research, and with the selected instruments (survey, teacher characterization test, semi-structured interview and class observations), relevant information was obtained for the objective pursued. It is intended to interpret in greater depth the teaching experiences in unitary schools and multigrade classrooms to develop and evidence processes of didactic transposition, with which, through the analysis of each teacher, their students can progress in the learning of plane and spatial geometry, in the particular transformation from the two-dimensional to the three-dimensional. Taking into account the result of the analysis of the information obtained during the 2014-2017 four-year period in eight rural institutions in different provinces of the department of Boyacá, with the support of the results of the SABER standardized tests of third and fifth grade of elementary school, the sense of relevance to strengthen the geometric-metric component and the competence of geometric reasoning was evidenced. These findings showed low averages of performance in geometry by students during the 2014-2017 four-year period.

Finally, some results obtained through the teacher characterization test applied in the doctoral research work in Education Sciences of the UPTC-RUDECOLOMBIA are detailed. This analysis revealed the main problems faced by teachers in these classrooms, in the department of Boyacá, but extrapolated to the entire country. In addition, the interpretation of these results provides tools to strengthen the teaching practices of the geometric-metric component. By articulating the quality references, the curricular updating references and Van Hiele's geometric reasoning model, it is intended to enable an adjustment in the planning processes carried out by educators in the rural sector of the department. This is presented as one of the few elements of support and effectiveness in the development of student competencies.

Methodology

Background and geographical delimitation

Figure 1

Map of Boyacá, with the location of the municipalities where the research was carried out

Source: Map of the Agustín Codazzi Geographic Institute, Colombia. Research data. In original Spanish language

In the classrooms selected for the study, the average is 16 students per grade, their economic condition is stratum one and two, it should be noted that most educators guide different areas of knowledge, highlighting mathematics (geometry); They are higher-level normalistas, with professional studies at the elementary level, mostly with specializations and some with master's degrees. They have an average of more than fifteen years of professional experience. The schools are unitary, that is, a single teacher guides the different grades of each site (transition, first, second, third, fourth and fifth) or multigrade classrooms where they have two or more teachers who work in the six grades with a greater number of students.

Consequently, the population and context are teachers of official primary school educational institutions with characteristics of multigrade and unitary classrooms in the rural sector, in the department of Boyacá, taking into account the municipalities of Chíquiza (I.E. San Pedro de Iguaque), Sotaquirá (I.E. José Joaquín Castro Martínez), Sutamarchán (I.E. Héctor Julio Gómez), Sáchica (I.E. Nueva Generación), Páez (I.E.T. José Antonio Páez), Tipacoque (I.E. Lucas Caballero Calderón), San Miguel de Sema (I.E. Técnica Alfonso Vanegas Sierra) and Sativanorte (I.E. Luis Manuel Parra Caro).

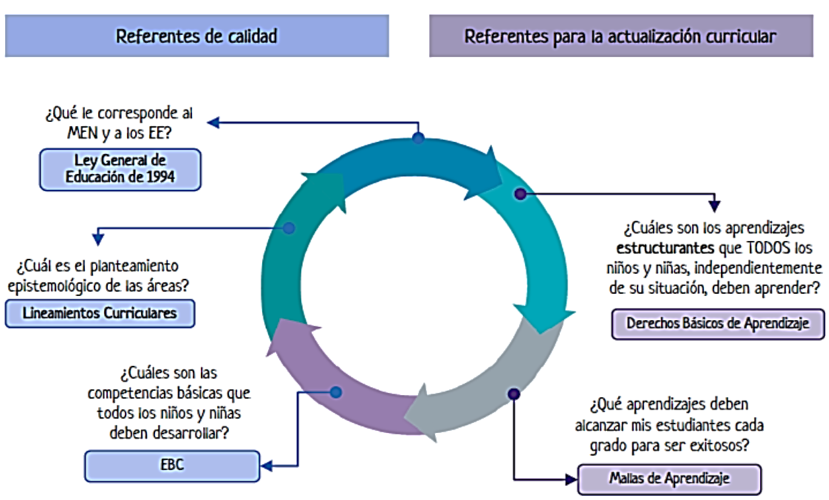
Curricular References in Colombia

Deficient geometric training in Latin America persists especially since the 1970s, when its teaching was neglected, particularly in teacher training. According to Cantoral, reversing this situation has been difficult from the conceptual and methodological context, due to the preference for an eminently algorithmic algebraic approach, without practical applications and disconnected from real contexts. As a result, students have lost key skills such as visualization, representations, exploration, modeling, argumentation, and demonstration.(1995) (Aravena & Caamaño, 2013)

Since the nineties, seeking to transform the educational system in Colombia, attention has been focused on this problem, seeking to strengthen the educational process of geometry by addressing primary education and incorporating their general, specific and content knowledge into a continuing education of teachers. Thus, agreements are reached from (MEN), taking into account what must be taught to read and write at the different educational levels. Figure 2 below presents the main tools available from the public policies of the MEN, while Figure 3 shows the timeline in which they were published.

Figure 2

Reference documents. In original Spanish language



Source: Authors.

To understand Figure 2, it is necessary to understand that, with the publication of the Political Constitution of 1991, the space was opened for the first educational reform in the country through the General Education Law. That is to say, for three decades the Colombian community of mathematics educators has been researching, reflecting and debating on the mathematical training of children and young people and on the way in which it can contribute more effectively to the goals and purposes of current education. This is how an interdisciplinary group addressed the problem and for the particular case of Mathematics, the Curricular Guidelines of 1998 appeared, which give a first horizon and reflect what the country's public policy intends to develop in each educational institution. Then in 2006 the Basic Competency Standards (EBDC) appeared, taking into account that they organize training cycles and the Mathematical activity, which will be worked with five (5) thoughts and five (5) systems that allow its development, these are: numerical thinking and numerical systems, spatial thinking and geometric systems, random thinking and data systems. as well as variational thinking and algebraic and analytical systems.(Ley 115 de 1994)(Ministerio de Educación Nacional, 2006)

The EBDCs propose and reflect the vision proposed from the Curricular Guidelines, which already proposed the transition towards the mastery of competencies by incorporating a pragmatic and instrumental consideration of mathematical knowledge, in which mathematical concepts, propositions and structures were used as effective tools, through which certain types of logical and mathematical thinking are put into practice. inside and outside the educational institution.(Ministerio de Educación Nacional, 2006)

It is therefore considered that the competencies of the area of mathematics are not developed taking into account spontaneous generation, on the contrary, they need educational environments enriched with contexts and problems that signify and yield a good understanding on the part of the student, which allow them to evolve in competencies with a greater degree of complexity. Thus, in the general process found in the curricular guidelines, they are structured from diagnosing and solving problems, as well as exemplifying processes and phenomena of the real context, communicating, reasoning, formulating, comparing and carrying out exercises related to algorithmic procedures.

With these approaches and new perspectives on mathematics education, which consider personal, cultural, social and political ends, new and challenging challenges arise. These challenges open horizons and justify the need for mathematical training in accordance with the purposes of education.

Figure 3

Chronological appearance, timeline for curricular references in Colombia since the issuance of the general education law, law 115 of 1994. In original Spanish language

Source: Authors. In original Spanish language

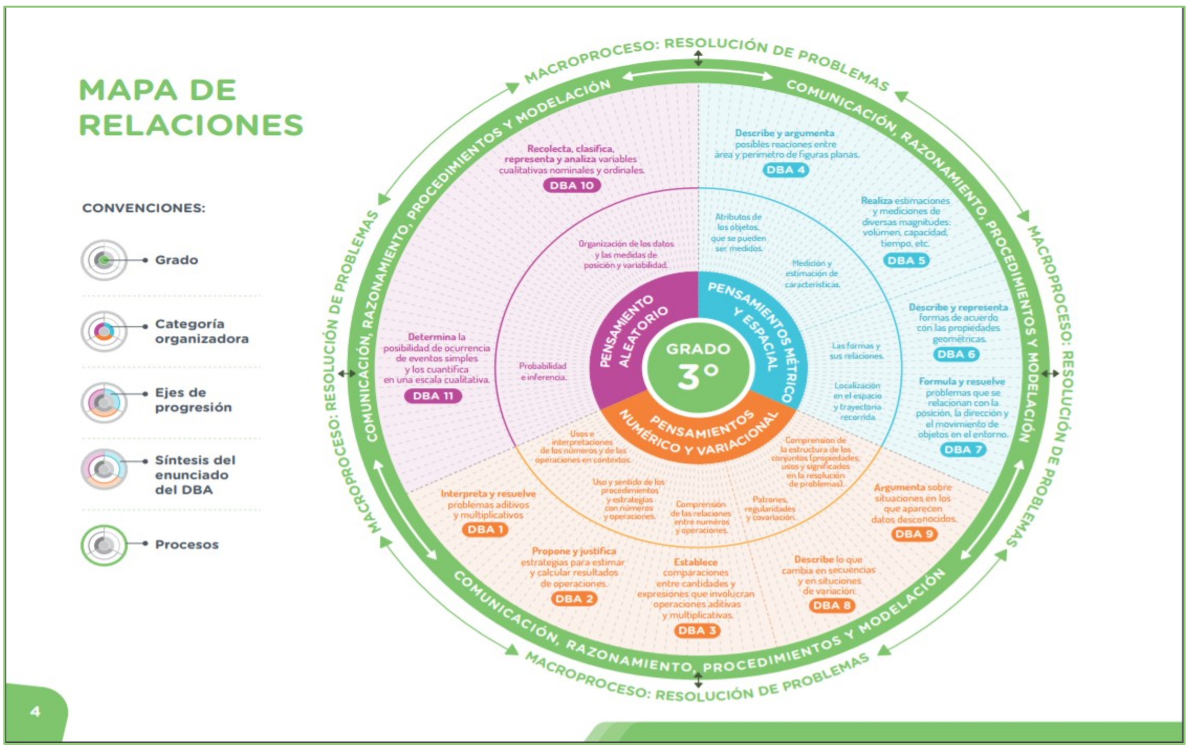
Starting in 2015, the Ministry of Education raised a concern about informing parents about what their children should learn in each of the educational grades, from first to eleventh grade. These publications gave rise to the 7 Basic Learning Rights (DBA), version 1, which were published in the newspaper El Tiempo and became the first reference for support or curricular updating. Subsequently, version 2 of the Basic Learning Rights was published, extended to different areas of knowledge and to the Transition degree in early education.

In 2017, the Learning Meshes were socialized in the city of Cali, which break down the DBAs and, through a progression of learning, show the content that the student must acquire in each of the educational levels, this in order for such learning to be successful. These meshes include a map of relationships that facilitates the planning process of educators, presented in a circular and radial diagram that shows the level, the category, the axes of progression, the synthesis of the DBA statement and the respective processes. Unfortunately, in the case of Mathematics, the meshes were only developed for elementary school.

Of these two instruments, only DBAs were introduced with any success to the teaching planning processes, or at least they are required, with very little knowledge of them.

Figure 4

Relationship Map



Source: Third grade mathematics learning meshes, adapted by the author. In original Spanish language

Finally, there is an instrument that, although it is not from the Ministry of National Education, accounts for the learning that the SABER tests evaluate in the third, fifth and ninth grades, called Reference Matrices, this document very little worked on by teachers in their planning exercise, it was developed by the Colombian Institute for the Evaluation of Education (ICFES). which is responsible for the design and application of these tests in the degrees mentioned above. Its organization is as follows:

Figure 5

SABER Test Reference Matrices

COMPETENCIES	DESCRIPTION	COMPONENTS	DESCRIPTION
COMMUNICATION	Refers, among other aspects, to the student's ability to express ideas, interpret, use different types of representation, describe mathematical relationships, etc.	**NUMERICAL-VARIATIONAL**	Refers to aspects related to numbers and numeration, their meaning, and the structure of the number system, among others.
REASONING	Related to the ability to understand how and why certain paths are followed to reach conclusions, etc.	**GEOMETRIC-METRIC**	Related to the construction and manipulation of representations of spatial objects, the relationships between them, and their transformations, etc.
PROBLEM SOLVING	Related, among other things, to the ability to formulate problems from situations inside and outside of mathematics, develop and apply different strategies, and justify the choice of methods and tools for problem-solving, etc.	**RANDOM**	Refers to the representation, reading, and interpretation of data in context.

Source: ICFES.

That is, he grouped the five thoughts and the five competencies of the Mathematical activity as follows:

Components: This collects the 5 thoughts and systems in three groups.

Variational numerical component: It refers to how the student understands numbers and what refers to numbering, in the same way it covers the meaning of number, as well as the order of numbering, taking into account the meaning of the operations, together with understanding the properties, the effect of a relationship that arises between them, due to the implementation of numbers and different operations in the solution of different problems, as well as the recognition of regularities and the referents that are used, the identification of variables, the description of the phenomena of change and dependence, this due to the criteria and procedures linked to direct variation, proportionality, as well as linear variation in the arithmetic and geometric context, to the inverse variation and finally to the end of the function. Geometric element of measurement: It refers to the creation and use of representations of the elements that air them, the link between them and the changes that are found, mainly from understanding space, the development of visual thinking, as well as the abstract analysis of figures from the plane and space by observing patterns, this is how geometric reasoning is used to solve problems in which measurement is concerned, the creation of concepts of a magnitude, and the understanding of definitions of perimeter, area, and volume.

Random: It focuses on investigating how information is represented, reading and comprehension, taking into account the diagnosis of different ways to represent numerical data, together with the qualitative examination of patterns, trends and types of growth, together with the elaboration of inferences and arguments using measures of central tendency and dispersion; in addition to the recognition, description and study of random events.

Competencies: The competencies of the mathematical activity were also collected or grouped into three groups. Taking into account that the tests evaluate different competencies in the area of mathematics from the concept of communication, modeling, reasoning, approach and problem solving, as well as the construction, comparison and exercise of procedures. It can then be said that, in the creation of the tests, these competencies are reordered as follows: reasoning and argumentation: communication, representation and modeling; the formulation and resolution of problems, in which they are immersed, the elaboration, comparison and exercise of procedures.

Reasoning and argumentation: It is related to fundamental aspects such as exemplifying the how and why of the methods used to reach conclusions, as well as supporting the different methodologies used in problem solving, formulating theories, making speculations, investigating, testing and constructing arguments, as well as globalizing properties, identifying patterns and expressing them mathematically. to ask questions, to recognize different kinds of reasoning, as well as to distinguish and evaluate chains of arguments.

Communication, representation and modeling: It refers to aspects such as how far the student is able to go to evidence ideas and interpret and use representative classes, describing mathematical associations and relating physical elements and diagrams with mathematical ideas, modeling a written, assertive, centered language that addresses symbology, as well as interpreting formal language, reaching the formal symbolic.

Problem Statement and Problem Solving: These aspects are related to the ability to formulate problems from situations inside and outside mathematics, to create and apply multiple strategies, together with justifying the choice of methods and instruments for problem solving, justifying the relevance of an exact or approximate calculation in the resolution of a problem, evaluate the reasonableness of an answer obtained, verify and interpret results in relation to the original problem, and generalize solutions and strategies to solve new problematic situations.

With the above, it can be seen that it is necessary to incorporate both quality and curricular support references into the planning processes carried out by teachers and to continue working on them as a possible way to strengthen the results of the area of study.

Geometric Reasoning

The concept of geometry for the Ministry of National Education of Colombia has an extensive history, which is linked to different human, social, cultural, scientific and technological activities. Moreover, geometry considered as the science that shows spatial reality, as an example of a formal system, or as a group of interrelated theories, is constantly changing and evolving, and cannot be limited only to formal propositions related to definitions, concepts, or theorems. (2004)

Geometric reasoning refers to the evaluation of the individual's processes and capacities in order to execute activities that need to observe or think about spatial geometric objects, as well as to involve elements and construct certain operations

or geometric transformations with them. Gil and De Guzmán show that geometric thinking consists of the cultivation of the parts of mathematics that try to stimulate man's ability to rationally explore his physical space, figure, and physical form.(1993)

To work on geometric thinking, skills are required, such as visually recognizing the shape of objects, consciously exploring space, comparing observed elements, establishing relationships between objects and verbalizing them, discovering properties of figures and transformations, and building models. (Van Hiele, 1957)

The observation of everyday problems that arise in the classroom with geometry students led professors Pierre Val Hiele and Dina Van Hiele-Geldof to carry out this work where they describe a scale with five hierarchical levels of the complexity of geometric thinking: how one thinks, what kind of ideas one possesses and how they are refined as one advances in the process from one level to another. This model covers two basic aspects: the descriptive one, through which different forms of the person's geometric reasoning are identified and can assess their progress (the levels), and the instructional one, through which guidelines are set for teachers, in order to observe and promote the progress of students at the level of geometric reasoning in which they are (the phases). (1957)(Vargas & Gamboa, 2013)

The levels are sequential, they are not associated with the age of the student, that is, they are not Piagetan, they depend on the teachings they have received and their experience, so it varies from one person to another. For further clarification, it may happen that a university student, for example, is at a lower level than a secondary or primary student, that is, his geometric knowledge is very low.

Table 1

Van Hiele Model of Geometric Reasoning

Descriptive	**Instructional**
LEVEL 1: Visualization or recognition	Phase One: Questions/Information
LEVEL 2: Analysis	Phase Two: Targeted Guidance
LEVEL 3: Sorting or Classification	Phase Three: Explanation
LEVEL 4: Formal Deduction	Phase Four: Free Orientation
LEVEL 5: Rigor	Phase Five: Integration

Source: Van Hiele described by Jaime y Fouz and De Donosti .(1957)(1993)(2005)

In their study, Vargas and Gamboa cite this model, which is described by Jaime and Fouz and De Donosti, in which the (2013)(1993)(2005)*Levels* They are named from one to five and are named as follows:

At the first level, *Visualization or recognition*, the concepts are considered globally, elements, properties or attributes are not taken into account. There are no characteristics of figures and descriptions of objects are made by visual perceptions.(Van Hiele, 1957)

At level two, *Analysis*, concepts are handled through their properties, the student identifies and generalizes properties of a concept, but does not establish relationships between them. Thus, students do not make inclusive classifications, because when defining concepts they list some necessary properties, others that are not and omit some essential ones; In addition, all discoveries or verifications are made using experimentation. (Van Hiele, 1957)

At level three, *Ordering or Classification*, students establish relationships between properties. Through experimentation, they verify to give results and accept new definitions of previous concepts. (Van Hiele, 1957)

Level four, *Formal deduction*, is in which formal demonstrations are made, linking simple implications, going from hypothesis to thesis, accepting the existence of equivalent definitions and alternative demonstrations. Finally, at level five, (Van Hiele, 1957)*Rigor*, the existence of different axiomatic systems is known.(Van Hiele, 1957)

The second part of Van Hiele's model is the *Phases*, which allow activities to be organized according to the model; These are: Beginning to inquire or information, directed orientation, explanation, free orientation and integration, as specific strategies for learning geometry. Following the same authors:(Vargas & Gamboa, 2013)

In *phase one* , questions and information are used, contact is made with the new topic or object of study. This is how the teacher identifies the previous knowledge and the degree of reasoning that the student may have regarding the topic being addressed. The student must receive information regarding the field of study that is going to be started, together with the types of problems that are going to be solved, the methods and materials that will be implemented.

In the *Phase Two*, explanation and demonstration are used. Students should be guided through activities, so that they discover and learn the various relationships or basic components of the knowledge to be formed. Jaime says that this phase is essential since the basic elements of the network of the corresponding level are built in it. Jaime adds, quoting Van Hiele, that "the activities of the second phase, if carefully selected, constitute the adequate basis for higher-level thinking" (p. 10).(1993)(1993)(1957)

For their part, Vargas and Gamboa state that "careful planning of the work sequence will take into account the need to achieve small successes that stimulate the student's self-esteem and favor a positive attitude towards mathematics" (p. 36)(2013)

In the *Phase Three*, Explanation, students exchange their experiences, comment on what they have done in class and justify their points of view according to what they have done. Likewise, they must combine and apply the knowledge acquired in the previous phases, to solve new activities (free orientation and integration), which corresponds to the following phases: four and five.(Vargas & Gamboa, 2013)

According to Fernández-Nieto, to avoid possible confusion, Van Hiele's model has the following characteristics: (2018)*Sequentiality*, it is not possible to alter the order of the levels for their acquisition; *Language specificity*, each level has its own language; *Move from one level to another*, it is not done abruptly, between one level and another there is a period in which reasoning of the two levels is presented, it is an intermediate state; *Globality and Locality*, the level of reasoning is local, that is, the level of reasoning for a given concept may vary for another; (Fernández-Nieto, 2018)*Instruction*, the acquisition of the levels is not biological, they involve the knowledge and personal experience of the student. (Camargo, 2011)

Moreover, Adulyasas and Abdul argue that a student develops an understanding of geometry, progressing through these five levels in a hierarchical manner. Studies carried out in Thailand with this method have shown that geometric thinking and their attitude towards the area improved. However, another study has shown that teachers tend not to use this method in the classroom due to time and do not want to use complex lessons, but stick to the traditional.(2014)(Ávila, 2017)(Canals & Solís, 2005)

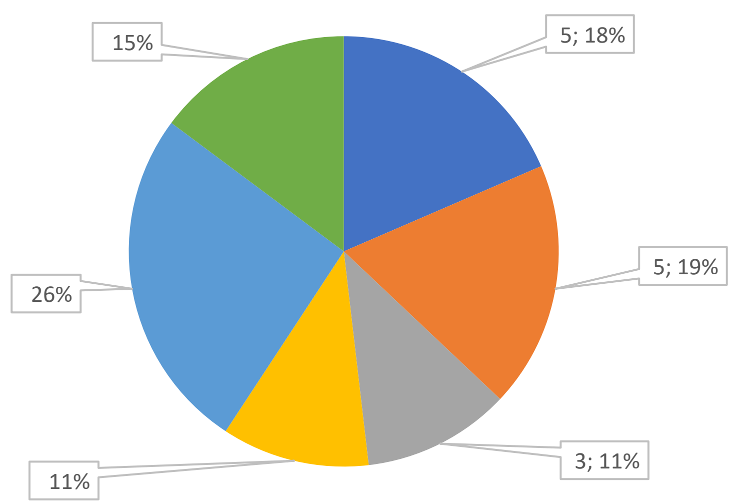
Duval states that, according to the teaching and learning of geometry, at least three cognitive activities are involved: construction, visualization and reasoning. The first (construction) refers to the design of configurations with geometric instruments; the second or visualization that indicates the spatial representations and the third (reasoning), which refers to the discursive processes used. In addition, he assures that each activity has different epistemological functions; for example, visualization allows the illustration of propositions, subjective verifications, the heuristic exploration of complex situations and synoptic views of them.(2001)

Results

Teacher Characterization Test

Figure 6

Types of questions used in the research, according to the level of acquisition (minimum, satisfactory and advanced)



Source: Authors.

Table 2

Answers to questions taking into account the level of acquisition, for the third grade

Third Grade

Minimum: 5 questions			Satisfactory: 5 questions			Advanced: 3 questions		
He said.	**Negat.**	**Total**	**He said.**	**Negat.**	**Total**	**He said.**	**Negat.**	**Total**
4 (100%)		4	4 (100%)		4	1 (100%)		1
1 (75%)	1 (25%)	1	1 (75%)	1 (25%)	1	2 (87.5%)	2 (12.5%)	2
		5			5			3

Source: own elaboration

From Table 2 it can be inferred that, from the third grade, thirteen (13) questions were taken, of which five (5) were of minimum acquisition level, five (5) of satisfactory and three (3) of advanced. The answers were: for the minimum level, four (4) answered affirmatively with 100% each and one with 75%; Another question (1) was answered negatively with 25%.

For the satisfactory level, four (4) answered affirmatively with 100% (this means that the eight teachers answered these questions with this percentage) and one (1) question with 75%. Negative answered at this satisfactory level, one (1) question with 25%.

Finally, for the advanced acquisition level, one question was answered affirmatively with 100%, and two (2) questions with affirmative answer each with 87.5% and the same two (2) questions with negative answer, each with 12.5%.

Table 3

Answers to questions taking into account the level of acquisition, for the fifth grade

Fifth grade								
Minimum: 3 questions			**Satisfactory: 7 questions**			**Advanced: 4 questions**		
Affirm.	**Denied.**	**Total**	**Affirm.**	**Denied.**	**Total**	**Affirm.**	**Denied.**	**Total**
3 (87.5%)	3 (12.5) c/u	3	3 (100%) 2 (87.5%)	2 (12.5%) c/u	5	1 (100%)	1 (100%)	2
			2 (75%)	2 (25%) c/u	2	2 (75%)	2 (25%)	2
		3			7			4

Source: own elaboration

From Table 3, it can be inferred that, from the fifth grade, fourteen (14) questions were taken, of which three (3) were of minimum acquisition level, seven (7) of satisfactory and four (4) of advanced. The answers were: for the minimum level, three (3) answered affirmatively with 87.5% each and another three (3) with 12.5% each in the negative.

For the satisfactory level, three (3) answered affirmatively with 100% (this means that the eight teachers answered these three questions with this percentage), two (2) questions with 87.5% and two (2) with 75%. Two (2) questions with 12.5% each and another two (2) questions with 25% each answered negatively at this satisfactory level.

Finally, for the advanced acquisition level, one question was answered affirmatively with 100%, but there was also one (1) question with a negative answer for another 100%, this means that, in this question, none of the eight teachers got the answer right. Also, at this level there were two (2) questions with an affirmative answer with 75% and the same two (2) questions with a negative answer, each with 25%.

On the other hand, Kuder Richardson's formula (KR-20), used to estimate the internal reliability of instruments, shows that it is consistent in its ability to measure the underlying construct; in this case (0.70), it was possible to establish that the answers to the questions designed for teachers reach levels of geometric reasoning according to Van Hiele between I and II, (visualization and analysis), but have difficulty at level III (ordering and classification), as previously exposed. Regarding this last level of geometric reasoning, it should be said that, from the curricular references, it is intended that students in the fifth grade of elementary school achieve it, however, if the teacher has difficulties in its apprehension, it is difficult for him to develop it through his teaching practices with his students. This makes it clear that work must be done on disciplinary training processes that allow the development of it, first in teachers, and later, they do it with their students.

Discussion of results

Survey

It should be noted that this instrument made it possible to identify general aspects of training and experience of the teachers involved in the study, thus, as a starting point it focuses on the fact that none of the eight teachers has disciplinary training in the area of mathematics, this is extrapolated to what happens with the vast majority of educators in the department and in the nation.

In this regard, we can highlight a problem in itself that drags the educational system of our country and most Latin American countries, which is that most teachers who carry out their work in primary school, have a very general training, because they must respond to all areas of knowledge. This is evidenced through this instrument that, in addition, shows that no teacher has specialization or training in the specific area of mathematics.

Another important aspect that is evident is that most rural educators immersed in research have been developing their teaching practice for many years and that their training and teaching practice process was carried out before the 90s, when real attention began to be paid to the teaching processes. specifically Geometry, with the enactment of the national constitution of 1991, the General Education Law of 1994 and the work that is generated from public policies in education and the MEN with quality references, curricular guidelines and Basic Standards of Competencies, time in which most teachers had already completed their training processes.(Lineamientos curriculares, 1998)(2006)

Regarding their knowledge of geometry, although they claim that they know it, their praxis says the opposite, only some plan the classes, few use resources such as the geoplane, tangram, concrete material, the classes are traditional and the authority of the class falls or is exercised by the teacher, the same happens with the evaluation processes, which lack context and the quantitative continues to be privileged over qualitative evaluation; thus, the above agrees with what Shulman and Chevallard, among others, say. (1989) (1991)

Teacher characterization test

With this instrument, the research sought to address the problem of the teaching of Geometry in basic primary education in Colombia, in unitary schools and multigrade classrooms in rural areas of the department of Boyacá. Through the questions of the test, it was possible to establish that teachers manifest using some elements alluding to didactic transposition, but there is no concrete and real knowledge of their benefits.

In the process of the knowledge imparted, teachers have noticed a greater assimilation on the part of the students, corroborating that they apply this knowledge in the environment that surrounds them, in the practice and development of the work and in the result of the evaluations applied. On the other hand, half of the teachers have difficulties in handling certain knowledge and topics specific to geometry. The best answers to the 35 questions of the test by subject, to give an idea of the level of knowledge of the teachers about geometry, were: Solid bodies: 4 questions; Flat Figures: 3 questions; Quadrilaterals, Parallels and Perpendiculars, Polygons, Homotecia, Symmetries with 1 question; Isometric transformations reached 2 correct questions. However, in question 26, on this same topic, no teacher got it right. In question 31 of flat figures, 7 teachers did not answer correctly and in question 29 of solid bodies, 6 teachers did not answer correctly either. When doing the math, only 14 questions were 100% effective among the 8 teachers. A little less than 50%.

In summary, applying the Van Hijale scale, most of the questions were level I and II for the third and fifth grades. But there were also III and IV levels. In general, the responses of the first two levels were good; but in the III and IV level there were more failures. These results are in agreement with what some authors say about the qualities of teachers and their teaching practices, as well as the level of acquisition. (OCDE, 2004)(Cochran-Smith, Zeichner, & Fries, 2006)(Cantoral, 1995)

Conclusions

Rural classrooms, with their two modalities, unitary and multigrade (where this study was developed), present a particularity in relation to graduate schools, and that is that they require a greater degree of knowledge and training of the educator who works in them; because the teacher must have a specialty in their disciplinary training processes, more time for planning, preparation and didactic resources that enrich their work. However, the fact that there is no support material designed to address these particularities, added to the low training process of the geometry area, affect that the expected results of the learning process at the rural level are not achieved.

When observing the answers given by the teachers that have to do with their perception of the teaching of Geometry, they consider that it is an easy area to teach, although they do not have the necessary training for the development of the processes or the didactic tools that allow successful teaching practices.

In this regard, it is observed that most teachers conceive the teaching of Geometry as the recognition of plane figures and spatial "figures" such as balls, or when drawing a ball (sphere) on the plane (board), they make a circle, unaware that from there a negative transpositive process is being generated by making plane geometry coincide and not differentiate from spatial geometry. In addition to the above, the didactic strategies used are used, where elements are used that determine the shapes of objects (windows, trusses, doors, desks, clocks to classify angles, etc...).

Some didactic resources that arrived at these institutions through a trunk, called Jaibana, which contained elements and didactic and pedagogical resources with specific material such as tangrams, Cuisenaire strips, geoplanes and other elements, remain stored in some institutions or if they are used they are not used to their potential, due to ignorance of their use.

At the same time, studies agree that learning geometry plays a fundamental role in problem solving, since, if it is considered as a model of representation and description of reality, it constitutes an important unifying element. From the point of view of learning, it helps to develop various reasoning processes, which in most cases have adequate graphic and manipulative supports, aspects that teachers carry out in their praxis.(Fortuny & Giménez, 1998)

The problem of teaching geometry, although it has been dragging on since the 1970s, began to be relevant in the nineties, although with incipient and generally endemic efforts. In addition to the above, the fact that most educators have not been able to introduce into their teaching practices references and curricular supports that have been generated by the MEN and the

ICFES; on the contrary, their practices are traditionalist (evidenced in class observations) and have not achieved an effective development of the subject of study.

In particular, the study focused on the cognitive process was analyzed, which is focused on how the development of competencies of geometric concepts is carried out, in order to explain the evolution of geometric thinking, focusing on the key processes, referring to visualization, recognition, identification, classification and representation, regarding which there are shortcomings in the teacher involved in the research.

The study was structured, taking into account the research of Gutiérrez and Jaime, who presented a more in-depth study regarding the description and acquisition of the student's reasoning, by transversalizing the levels of reasoning of the Van-Hiele with differentiated arguments of the reasoning processes for each of the levels. When conducting the study, what Van Hiele considered was applied to the teachers. The processes considered were: Characterize and describe the geometric objects, observable at levels 1 and 2; use of definitions, from level 2 to 4; formulation of definitions from level 1 to 4; classification of definitions from level 1 to 3 and observable demonstration in the productions of teachers from level 2 to 4. This form of analysis is neither contradictory nor exclusive of the proposals of Vinner and Hershkowitz and Presmeg, with respect to visualization, geometric processes, and the construction of the object.(1998)(1983)(1986)

The areas that were articulated in this research correspond to the levels of reasoning of the Van-Hiele model and the distinctive attributes in the reasoning processes for each of the levels, proposed by Gutiérrez and Jaime. These processes are those that are referenced with the aim of identifying the degree of reasoning in which the teaching staff is located.(1998)

By individualizing the degree of reasoning of the educators in these classrooms, it is defined that in geometry, most of the teachers have an acquisition with respect to levels 1 and 2 and contain deficiencies of grade 3. By not knowing this aspect in its levels and phases, it is almost impossible to support the geometric reasoning processes of its students.

There are serious deficiencies with the didactics of the knowledge designated from the public policies of the MEN, to the knowledge that is finally taught in the classrooms, the above encompasses multiple factors that range from the same processes of teacher training in the faculties of education, the scarce epistemological surveillance that is carried out on the textbooks that are used in the teaching and learning processes, the lack of disciplinary specialization to develop the processes that allow us to understand how to acquire such knowledge, etc.

Thus, through the instruments, teachers state that they apply different processes, but none comes close to what the didactic transposition process really is, in which the theoretical context to be taught must be characterized, as well as analyze the learning context and adapt theoretical knowledge to the context. establishing clear learning objectives and selecting assertive strategies to evaluate learning.

On the other hand, they say that didactic transposition is the adaptation of knowledge for teaching, they plan their classes using various strategies such as planning, audiovisual resources, elaboration of workshops and games, in addition to the creation of environments that are didactic in order to encourage the learning and interest of students; In other aspects, some teachers have difficulties in handling certain knowledge and topics specific to geometry, which prevents them from transmitting knowledge in an ideal way to students. Therefore, these educators do not meet the level of performance, although it is meritorious that they recognize that they do not have the skills to handle a geometry class, which is in accordance with what Chevallard, Aroca and Shulman say.(1991)(2019)(1989)

It is necessary to promote training for teachers in processes that have to do with geometry because of its importance for school-age children, thinking about their future life. They must deepen the ability to manage the geometry class, making it more complete, in order to ensure that students assimilate the knowledge and apply it in their daily lives, because it is not enough to do a playful class, or to make a lot of effort, because you do not have disciplinary mastery. The processes used in didactic transposition are contextualization, practice, applicability and search for various tools that adapt to the reality of each student. For the evaluation, they ask the students about the topics explained in class and carried out in the activities and evaluation processes.

Another factor that helps to understand this problem refers to the interaction that is found in the classroom. According to the teachers in the study, it is good, which is why what Osorio, Vidanovic and Finol say is true, who state that "between teachers and students they maintain a two-way relationship that impacts the teaching-learning process"; but also, another study carried out by Villalta, Martinic and Guzmán affirms that, for the most part, the exchanges that take place in the classrooms between the student and the teacher are of low cognitive demand. The exchanges are of the question-answer type and of induction of answers, where the teacher offers clues to the students so that they deliver the answer expected by the latter, and in the case of didactic interaction, the teacher is the protagonist of the process (traditional education continues), despite the great diffusion that has been made to new methods (e.g. constructivism) and other emerging activities. such as computers, especially for geometry (Cabri, Geogebra and others) and other resources (geoplane, tangram), etc.(2021, p. 11)(2011)

The survey and the semi-structured interview allowed us to know closely aspects related to the training of the educators who participated in this research, most of whom have been working since long before the educational reforms in Mathematics began in our country, or had already completed their training processes. Despite the fact that from 2015 to 2017 some curricular support references were generated by the MEN and ICFES, these have not managed to impact all the institutions of the department to advance curricular updating processes that allow appropriating these references to their planning processes.

On the other hand, education in our country shows an accentuated hierarchization and stratification, which considers the teacher of rurality and the basic primary education level at the lowest point of the pyramid, this issue of vital importance has been analyzed in other more developed countries in the educational issue, inverting the pyramid and highlighting the importance at the beginning of the training processes of students, placing the teachers who have the highest degrees of training in the initial teaching processes.

Another aspect of great importance is the fact that most teachers finished their training process (very general) and began their work long before public policies in education paid some attention to the teaching of Geometry and there has been little support offered for teacher training and updating. This is evidenced in the instrument in which teachers do not have disciplinary appropriation, didactic strategies to carry out planning processes according to the requirements of the new educational reality. It is so much so that, among the few attempts to meet this objective, they appear with the PER in 2012 and in 2013 with the Todos a Aprender Program, which through the accompaniment in situ and work of academic peers manages to impact a significant percentage of institutions in the department and the country. However, they are incipient and scarce due to bureaucratic and administrative procedures of a national nature that prevent continuity and permanence over time.

It is argued that the problem of geometric formation began in the seventies, mainly in Latin America, favoring an eminently algorithmic algebraic work, without applications and far from contexts. That is why it is true that students lose skills that are considered key in geometric processes, such as visualization, representations, exploration, modeling, argumentation and demonstration. (Aravena & Caamaño, 2013)

However, with all this, it has not been possible to overcome the weaknesses that have been pointed out. There are several studies that justify the reasons for these difficulties, among which the way in which it is taught, in teacher training, with a formal approach, overloaded with structuralism, abstraction and fragmentation of knowledge, stands out. It is evident that this form of teaching, which teachers acquire, tends to be reproduced in the classroom, generating in students little understanding of geometric concepts and processes. On the other hand, studies along these lines present two elements that could shed light on understanding these difficulties. The first is the little time dedicated to the subject, generally taught at the end of the courses or what is known as the school year, so there is little time left for teaching and learning; the second is that textbooks include the topics at the end of the chapters, which contributes to them not being considered by teachers. (OECD, 2004) (OCDE y Banco Mundial, 2009)(Latorre, 2004)(Aravena, Caamaño, González, Cabezas, & Córdova, 2011) (Blanco & Barrantes, 2003)(Gómez, 2000)

Finally, another aspect that is evidenced corresponds to what most of the participants say about their teaching experience, which despite being extensive in time, is scarce in depth, because due to the work of this type of classroom, the teacher has been relying on school textbooks that were introduced to the institutions through the adoption of the Escuela Nueva program. long before the importance of geometry as a subject of study in the teaching cycles was considered.

For these classrooms where educators must attend several or all courses or levels of training in all subjects, there are no textbooks that allow the sequential development of learning; this means that, although the teacher is within his schedule developing the subject of Geometry, he does not have the resource (textbook) or didactic sequence that allows him to work on it at all levels of basic training; clarifying, in the only classroom of the school, the teacher works two, or three courses if it is multigrade; if it is unitary all the courses-; Each child, in each course, has his or her new school card, works wherever he or she goes (he or she is autonomous), the teacher passes by and reviews his or her activity, stopping where someone asks or requires an explanation, and thus, in this way, work is done in unitary schools or multigrade classrooms.

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